DETERMINING THE THERMAL CONDUCTIVITY OF THIN-LAYER MATERIALS BY THE INTERPOLATION METHOD

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The interpolation method is outlined, by which the thermal conductivity of thin-layer materials can be determined without measuring the specimen thickness and with the effect of thermal contact resistances eliminated.

The main source of errors in the thermophysical testing of thin-layer materials is the contact resistance, which may be of the same order of magnitude as the thermal resistance of the specimen itself. When a specimen is very thin, furthermore, the magnitude of the relative error in its thickness measurement may become appreciable, especially in the case of specimens not quite homogeneous throughout their thickness.

A method will be described which, first of all, eliminates the need for measuring the specimen thickness and, secondly, eliminates the effect of thermal contact resistances.

A test plate was placed in a vessel containing a liquid with a known thermal conductivity λ_1. The thickness of the liquid layer, i.e., the depth of the vessel was either equal to or slightly greater than the maximum thickness of the immersed specimen. This thickness h could be measured with sufficient accuracy. The "effective" thermal conductivity λ_1' of the liquid–specimen system was also measured. Then, after the specimen had reached the state of equilibrium with the ambient medium, the effective thermal conductivity λ_2' of the system was measured with the same specimen immersed in another liquid with the thermal conductivity λ_2.

The true thermal conductivity of the test plate is found as follows. The values of λ_1 and λ_2 are marked on the axis of abscissas, while the effective values λ_1' and λ_2' are marked on the axis of ordinates (Fig. 1). If no specimen were placed in the vessel, then $\lambda_1' = \lambda_1$ and $\lambda_2' = \lambda_2$, i.e., these points would lie on a straight line passing through the origin of coordinates at a 45° angle to the axis of abscissas (with the same scale on both axes). The presence of a specimen in the vessel makes $\lambda_1' \neq \lambda_1$ and $\lambda_2' \neq \lambda_2$. Consequently,

TABLE 1. Relation $M^2 = f(\theta)$

θ	0,50	0,55	0,60	0,65	0,70	0,75
M^2	2,56	3,47	4,84	7,40	11,20	17,64

TABLE 2. Relation $\tau = F(\theta)$

θ	0,50	0,55	0,60	0,65	0,70
τ, sec for celluloid in oil	19,2	27,4	39,5	59,6	91,6
τ, sec for celluloid in water	15,7	22,4	32,0	46,8	71,0

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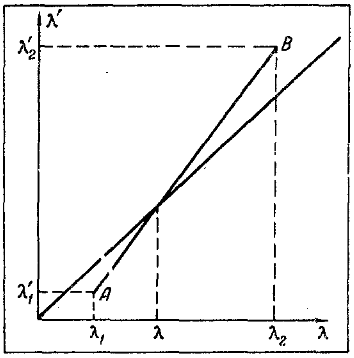

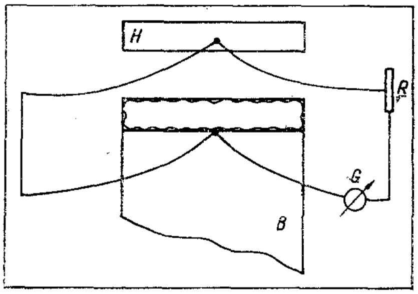

Fig. 1. Graph of the relation $\lambda' = f(\lambda)$. Thermal conductivities λ' and λ (W/m·°C).

Fig. 2. Schematic diagram of the test setup.

the intersection of the straight line through points A and B with the straight line inclined at 45° yields the desired value of λ.

The relation $\lambda' = f(\lambda)$ is represented by a straight line (as has been verified on several points, with different liquids) and may be expressed in terms of the equation

$$\lambda' = m\lambda + n. \tag{1}$$

With the values of λ_1, λ_2, λ_1' and λ_2', it is not difficult to find the desired thermal conductivity λ by the formula

$$\lambda = \frac{n}{1-m},$$

where

$$m = \frac{\lambda_2' - \lambda_1'}{\lambda_2 - \lambda_1} \quad \text{and} \quad n = \lambda_1' - m\lambda_1 = \lambda_2' - m\lambda_2.$$

For illustration, we determined the thermal conductivity of a thin celluloid plate. The plate was not quite homogeneous throughout its thickness and the latter was not measured. The effective thermal conductivities λ_1' and λ_2' were measured by the method shown in [1].

According to the same method we measured one time interval $\Delta\tau = \tau_2 - \tau_1$ corresponding to a change $\Delta\theta = \theta_2 - \theta_1$ in the relative temperature. Here $\theta_1 = 1 - N_1/N_0$ and $\theta_2 = 1 - N_2/N_0$ with N_1, N_2 denoting any arbitrary divisions on the galvanometer scale G convenient for the measurements (Fig. 2) and N_0 denoting the division on the galvonometer scale prior to the beginning of the test, i.e., before heater H had established contact with the system.

The thermal conductivity is found here by the formula

$$\lambda = \frac{bhM}{2\sqrt{\Delta\tau}} = \frac{bh}{2\sqrt{\Delta\tau}}\sqrt{M_2^2 - M_1^2}, \tag{2}$$

where the constant b characterizes the thermal activity of heat receiver B and is determined by a calibration of the latter.

TABLE 3. Relation $\sqrt{\Delta\tau/M^2} = \varphi(\Delta\theta)$

$(\Delta\theta)$, %	70—65	70—60	70—55	70—50	65—55	65—50	
M^2	3,80	6,36	7,73	8,64	3,93	4,84	
$\Delta\tau$, sec	32,0	52,1	64,2	72,4	32,2	40,4	1st-test
$\sqrt{\frac{\Delta\tau}{M^2}}$, $\sec^{1/2}$	2,91	2,86	2,88	2,90	2,86	2,89	the same
$\Delta\tau$, sec	24,2	39,0	48,6	55,3	24,4	31,1	2nd-test
$\sqrt{\frac{\Delta\tau}{M^2}}$, $\sec^{1/2}$	2,53	2,48	2,51	2,53	2,50	2,54	the same

To every value of the relative temperature θ_i there corresponds a definite value of M_i. The relation $M^2 = f(\theta)$ is shown in Table 1. The relation $\tau = F(\theta)$ has been plotted for several values of θ, making it possible to use several temperature differences $\Delta\theta$ from which the mean values $\sqrt{\Delta\tau/M^2}$ for Eq. (2) could be obtained. The relation $M^2 = f(\theta)$ is valid for sufficiently large values of θ and for $\varepsilon = \lambda/b\sqrt{a} < 1$, where b is the heat receiver constant and a, λ are thermophysical properties of the test material. In order to satisfy the condition 1 in tests, one must use a heat receiver with a large constant b (marble, cement, etc). In our tests the heat receiver had been made of marble with a thermal activity, according to our measurements, $b = 2430\ W \cdot sec^{1/2}/m^2 \cdot °C$. The end surface of this heat receiver served as the bottom of the test vessel. In order to render the results of measurements independent of the contact point between the thermocouple junction and the system (with the liquid or with the specimen), the junction was separated from the system by a thin metallic foil $10 \times 10\ mm^2$ in area. The depth of the vessel was $h = 0.50$ mm. As reference liquids we used technical-grade oil ($\lambda_1 = 0.124$ W/m $\cdot$°C) and water ($\lambda_2 = 0.580$ W/m $\cdot$°C).

The results of measurements are shown in Table 2 and an evaluation of these two tests is given in Table 3.

From the data in Table 3 we find the values of the quantity $(\Delta\tau/M^2)^{1/2}$: 2.88 $sec^{1/2}$ for the first test and 2.515 $sec^{1/2}$ for the second test. From this we have

$$\lambda_1' = \frac{2430 \cdot 5 \cdot 10^{-4}}{2 \cdot 2.88} = 0.211\ \mathrm{W/m \cdot °C} \quad \text{and} \quad \lambda_2' = 0.241$$

With the values of λ_1, λ_2, λ_1', and λ_2' known, we find the coefficients m and n for Eq. (1):

$$m = 0.066 \text{ and } n = 0.203\ \mathrm{W/m \cdot °C}.$$

The desired thermal conductivity of celluloid is then

$$\lambda = 0.22\ \mathrm{W/m \cdot °C}.$$

The same value can be found from the graph in Fig. 1.

NOTATION

λ is the thermal conductivity;
λ' is the effective thermal conductivity;
t is the temperature;
t_H is the heater temperature;
τ is the time.

LITERATURE CITED

1. V. S. Vol'kenshtein and N. N. Medvedev, in: Heat and Mass Transfer [in Russian], Izd. Nauka i Tekhnika, Minsk (1968).